Math 128a – Week 2 Worksheet
GSI: Izak, (1/27/21)

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1 1.1 Problems

Problem 1. *Let $f(x) = x^3$, (1) find the second order Talyor polynomial $P_2(x)$ about $x_0 = 0$. Compute $P_2(x)$ and the error $R_2(x)$ (2) do the same but with $x_0 = 1$*

Problem 2. *Let $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$, integrate the Taylor series to show that*

$$erf(x) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)k!}$$

Problem 3 (Hard)**.** *If $f(x)$ satisfies $|f(x) - f(y)| \leq L|x - y|$ for all x, y and some fixed constant $L > 0$, prove f is continuous.*

Suppose further that $|f(x) - f(y)| \leq L|x - y|^\alpha$ for fixed L and $\alpha > 1$, show that f is constant[a]*.*

2 1.2 Problems

Problem 4. *Compute the following (1) exactly (2) using three-digit chopping arithmetic (3) using three-digit rounding arithmetic (4) find relative errors in (2) and (3)*

$$\left(\frac{1}{3} - \frac{3}{11}\right) + \frac{3}{20}$$

Problem 5. *Compute the relative error between $e = \sum_{n=0}^{\infty} \frac{1}{n!}$ and $\sum_{n=0}^{m} \frac{1}{n!}$ for $m = 5$ and do it again but for $m = 10$.*

Problem 6 (Hard)**.** *If $fl(y)$ is a k-digit rounding approximation to y, show that:*

$$\left|\frac{y - fl(y)}{y}\right| \leq .5 \cdot 10^{-k+1}$$

3 1.3 Problems

Problem 7. *Find the rate of convergence of the sequence as $n \to \infty$:*

$$\lim_{n \to \infty} \sin \frac{1}{n} = 0$$

Problem 8. *Find the rate of convergence of the sequence as $n \to \infty$:*

$$\lim_{n \to \infty} [\ln(n + 1) - \ln(n)]$$

Problem 9. *Find the rate of convergence of the sequence as $n \to 0$:*

$$\lim_{n \to 0} \frac{1 - \cos n}{n}$$

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[a] there's a funny story about this